

## Ableitungen von Funktionen: Lösungen

Bestimmen Sie die ersten und zweiten Ableitungen zu folgenden Funktionen

Aufgaben:

1.

$$f(x) = e^x$$

$$f'(x) = e^x$$

$$f''(x) = e^x$$

2.

$$f_1(x) = e^{x-3}$$

$$f_1'(x) = e^{x-3}$$

$$f_1''(x) = e^{x-3}$$

3.

$$f_2(x) = e^{-2x+7}$$

$$f_2'(x) = -2e^{-2x+7}$$

$$f_2''(x) = 4e^{-2x+7}$$

4.

$$f_3(x) = e^{-x^2}$$

$$f_3'(x) = -2xe^{x^2}$$

$$f_3''(x) = 2(2x^2 - 1)e^{-x^2}$$

5.

$$f_4(x) = e^{-5x^3}$$

$$f_4'(x) = -15x^2e^{-5x^3}$$

$$f_4''(x) = 15x(15x^3 - 2)e^{-5x^3}$$

6.

$$f_5(x) = e^{6x^2+2x-3}$$

$$f_5'(x) = (12x + 2)e^{6x^2+2x-3}$$

$$f_5''(x) = 4(3x^2 + 12x + 4)e^{6x^2+2x-3}$$

7.

$$f_6(x) = e^{\sqrt{2x-3}+6x}$$

$$f_6'(x) = \left( \frac{x}{\sqrt{2x-3}} + 6 \right) e^{\sqrt{2x-3}+6x}$$

$$f_6''(x) = \left[ \left( \frac{x}{\sqrt{2x-3}} + 6 \right)^2 - \frac{x^2 - 2x + 3}{\sqrt{(2x-3)^3}} \right] e^{\sqrt{2x-3}+6x}$$

1.

$$h_1(x) = \ln(x)$$

$$h_1'(x) = \frac{1}{x}$$

$$h_1''(x) = -\frac{1}{x^2}$$

2.

$$h_2(x) = \ln(2x)$$

$$h_2'(x) = \frac{1}{x}$$

$$h_2''(x) = -\frac{1}{x^2}$$

3.

$$h_3(x) = \ln(x^2)$$

$$h_3'(x) = \frac{2}{x}$$

$$h_3''(x) = -\frac{2}{x^2}$$

4.

$$h_4(x) = x \ln(x) - x$$

$$h_4'(x) = \ln(x)$$

$$h_4''(x) = \frac{1}{x}$$

5.

$$h_5(x) = \ln(x^2 - 5x + 1)$$

$$h_5'(x) = \frac{2x - 5}{x^2 - 5x + 1}$$

$$h_5''(x) = \frac{-2x^2 + 10x - 23}{(x^2 - 5x + 1)^2}$$

6.

$$h_6(x) = \ln(x^2 + e^{x^2})$$

$$h_6'(x) = \frac{2x(1 + e^{x^2})}{x^2 + e^{x^2}}$$

$$h_6''(x) = \frac{-2x^2 + 2(1 - 3x^2 + 2x^4)e^{x^2} + 2e^{2x^2}}{(x^2 + e^{x^2})^2}$$

7.

$$\begin{aligned}h_7(x) &= \frac{\ln(x)}{\ln(x^2 + x)} \\h_7'(x) &= \frac{\ln(x+1) \cdot (x+1) - \ln(x) \cdot x}{x \cdot (x+1) \cdot (\ln(x^2 + x))^2} \\h_7''(x) &= \frac{(1-x^2)(\ln(x+1))^2 + x^2(\ln(x))^2 - (2x+1)\ln(x)\ln(x+1)}{x^2(x+1)^2(\ln(x^2+x))^3} \\&\quad + \frac{2(2x+1)(x\ln(x) - \ln(x+1)(x+1))}{x^2(x+1)^2(\ln(x^2+x))^3}\end{aligned}$$

8.

$$\begin{aligned}h_8(x) &= \frac{x^2 - 1}{\ln(x)} \\h_8'(x) &= \frac{2x^2 \ln(x) - (x^2 - 1)}{x(\ln(x))^2} \\h_8''(x) &= \frac{2x^2(\ln(x))^2 - (3x^2 + 1)\ln(x) + 2(x^2 - 1)}{x^2(\ln(x))^3}\end{aligned}$$

9.

$$\begin{aligned}h_9(x) &= \ln\left(\frac{x+1}{x-1}\right) + \frac{e^x - e^{-x}}{e^x - 2} \\h_9'(x) &= \frac{2}{x+1} + 2\frac{1 - e^x - e^{-x}}{(e^x - 2)^2} \\h_9''(x) &= \frac{-2}{(x+1)^2} + 2\frac{3 + e^{2x} - 2e^{-x}}{(e^x - 2)^3}\end{aligned}$$